Credit Risk Simulation

Counterparty Credit Risk (CCR) Definition

- Counterparty credit risk refers to the risk that a counterparty a financial contract could fail to honor its obligation that causes financial loss to the non-defaulting party.
- Counterparty risk is present in financial market, especially in over-the-counter (OTC) derivatives and financial security transactions (FSTs) (e.g., repos).
- If one party of a contract defaults, the non-defaulting party will find a similar contract with another counterparty in the market to replace the default one. That is why counterparty credit risk sometimes is referred to as replacement risk.

Monte Carlo Simulation

- To calculate credit exposure or replacement cost in future times, one needs to simulate market evolutions.
- Simulation must be conducted under the real-world measure.
- Simple solution
  - Some vendors and institutions use this simplified approach
  - Only a couple of stochastic processes are used to simulate all market risk factors.
  - Use Vasicek model for all mean reverting factors

$$dr = k(\theta - r)dt + \sigma dW$$

where r – risk factor; k – drift; θ – mean reversion parameter; σ – volatility; W – Wiener process.

Monte Carlo Simulation (Cont'd)

- Use Geometric Brownian Motion (GBM) for all non-mean reverting risk factors.

$$dS = \mu S dt + \sigma S dW$$

where S – risk factor; μ – drift; σ – volatility; W – Wiener process.

- Different risk factors have different calibration results.

- Complex solution
  - Different stochastic processes are used for different risk factors.
  - These stochastic processes require different calibration processes.
  - Discuss this approach in details below.

Yield curve simulation

- Simulate yield curves (zero rate curves) or swap curves.
- There are many points in a yield curve, e.g., 1d, 1w, 2w 1m, etc. One can use Principal Component Analysis (PCA) to reduce risk factors from 20 points, for instance, into 3 point drivers.
- Using PCA, you only need to simulate 3 drivers for each curve. But please remember you need to convert 3 drivers back to 20-point curve at each path and each time step.

Yield curve simulation (Cont'd)

- One popular yield curve simulation model under the real-world measure is the Cox-Ingersoll-Ross (CIR) model.

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dW$$

where r – risk factor; k – drift; θ – mean reversion parameter; σ – volatility; W – Wiener process.

- Reasons for choosing the CIR model
  - Generate positive interest rates.
  - It is a mean reversion process: empirically interest rates display a mean reversion behavior.
  - The standard derivation in short term is proportional to the rate change.

Forex rate simulation

- Simulate foreign exchange rates.
- Black Karasinski (BK) model:

$$d(\ln(r)) = k(\ln(\theta) - \ln(r))dt + \sigma dW$$

where r – risk factor; k – drift; θ – mean reversion parameter; σ – volatility; W – Wiener process.

- Reasons for choosing BK model:
  - Lognormal distribution;
  - Non-negative FX rates;
  - Mean reversion process.

Stock price simulation

- Simulate stock prices.
- Geometric Brownian Motion (GBM)

$$dS = \mu S dt + \sigma S dW$$

where S – stock price; μ – drift; σ – volatility; W – Wiener process.

- Pros
  - Simple
  - Non-negative stock price
- Cons
  - Simulated values could be extremely large for a longer horizon, so it may be better to incorporate with a reverting draft.

Commodity simulation

- Simulate commodity spot, future and forward prices, pipeline spreads and commodity implied volatilities.
- Two factor model

$$\log(S_t) = q_t + \mathcal{X}_t + \mathcal{Y}_t$$
$$d\mathcal{X}_t = (\alpha_1 - \gamma_1 \mathcal{X}_t)dt + \sigma_1 dW_t^1$$
$$d\mathcal{Y}_t = (\alpha_2 - \gamma_2 \mathcal{Y}_t)dt + \sigma_2 dW_t^2$$
$$dW_t^1 dW_t^2 = \rho dt$$

  where S_t – spot price or spread or implied volatility; S_t – deterministic function; $\mathcal{X}_t$ – short term deviation and $\mathcal{Y}_t$ – long term equilibrium level.

- This model leads to a closed form solution for forward prices and thereby forward term structures.

Implied volatility simulation

- Simulate equity or FX implied volatility.
- Empirically implied volatilities are more volatile than prices.
- Stochastic volatility model , such as Heston model

$$dS_t = \mu S_t dt + \sqrt{\mathcal{V}_t} S_t dW_t^1$$
$$d\mathcal{V}_t = \kappa(\theta - \mathcal{V}_t)dt + \xi\sqrt{\mathcal{V}_t} dW_t^2$$
$$dW_t^1 dW_t^2 = \rho dt$$

  Where S_t is the implied volatility and $\mathcal{V}_t$ is the instantaneous variance of the implied volatility

- Pros
  - Simulated distribution has fat tail or large skew and kurtosis.

Implied volatility simulation (Cont'd)

- Cons
  - Complex implementation
  - Unstable calibration
- If a stochestic volatility model is too complex to use, a simple alternative is

$$dr = k(\theta - r)dt + \sigma dW$$

where r – volatility risk factor; k – drift; θ – mean reversion parameter; σ – volatility; W – Wiener process.

Thanks!

You can find more details at
https://finpricing.com/lib/IrCurveIntroduction.html